Comment on "'t Hooft vertices, partial quenching, and rooted staggered QCD"

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Abstract

A recent criticism of the proof of the failure of the rooting procedure with staggered fermions is shown to be incorrect.

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In a recent paper [1] Bernard, Golterman, Shamir and Sharp challenge the proof developed in [2, 3, 4] showing that non-perturbative effects are incorrectly treated in the rooting formalism popular for reducing the number of fermion species in the staggered formalism for dynamical quarks. Here I discuss how this challenge is based on a misunderstanding of the chiral behavior of staggered quarks.

The problem appears already in the introduction of [1] where the authors attempt to define something they call the "rooted continuum theory" (RCT). While technically the paper makes the qualifying statement "What is less certain ... is that the rooted staggered theory on the lattice becomes this RCT as $a \to 0$," the discussion misleadingly continues as if the RCT can be defined in two inequivalent ways. First they consider the continuum limit of staggered fermions treated with the rooting trick. But then towards the end of the introduction they say the RCT can be obtained rigorously from rooting four copies of a chirally invariant formulation, such as with the overlap operator [5]. For the latter theory the correctness of rooting is a trivial mathematical identity.

The point of the discussion in Refs. [2, 3, 4] is that these two approaches display qualitatively different non-perturbative effects. Only the latter form generates the correct one flavor theory. Confusing these theories is equivalent to assuming that rooting is correct and misses the issues that invalidate the rooting prescription when used with staggered quarks. Throughout the rest of the paper they make no distinction between these definitions, just referring to the RCT as the physical one flavor theory. If the RCT operator is chosen from rooting four equivalent copies of a properly defined chiral fermion theory, then the remaining discussion in Ref. [1] is simply a verification of the trivial reduction to the one flavor case.

The crucial issue is that after rooting the staggered propagator still represents four independent fermion states. A valid single fermion propagator would have only a single pole. While rooting does correctly reweight perturbative loops, it fails when instantons are present. Then the 't Hooft vertex [6] directly couples all fermion species, including any extra tastes. Rooting four powers of a true single fermion theory involves a propagator that has only one physical pole. Coupling four copies of this pole via instantons is impossible due cancellations from Pauli statistics. This cancellation does not occur for the staggered tastes which remain as independent states, leaving an incorrect form for the 't Hooft vertex. The undesired effects occur at a typical instanton scale, which is set by $\Lambda_{\rm QCD}$, and will survive the continuum limit. The problems appear whenever instanton physics is important, irrespective of

whether the quark masses vanish.

The introduction to Ref. [1] also propagates the misconception that it is only taste breaking and mixing that can cause problems. The issue with rooting is not taste breaking but the strong coupling between the tastes induced by these non-perturbative effects. The troublesome coupling takes the form of a determinant that is in fact taste symmetric. Since the coupling involves all tastes, it appears only in processes roughly comparable to four loops in the perturbative expansion. Thus these effects may not be large in flavor non-singlet processes. But their existence rules out the method as a first principle approach to physical observables.

That taste breaking and the strong coupling between tastes are independent issues has recently been explored in Ref. [7], where a two taste model for rooting is constructed with taste mixing explicitly removed. The arguments of Refs. [2, 3, 4] still apply, and rooting fails in this model also because the two tastes involved are physically inequivalent. The model is similar to Wilson fermions but considers one taste with an effective the strong interaction CP angle theta of $\pi/2$ and the second of $-\pi/2$, as discussed in Ref. [8]. This rotation allows a residual chiral symmetry to survive at finite lattice spacing, but does not commute with the rooting process. Considered as a two flavor theory, these phases cancel, but on rooting one is working with a mixture of two inequivalent one-flavor theories. Ref. [7] does not commit on whether this make sense, but argues that if it does, the theory will not have CP violation and therefore must be the one flavor theory at vanishing theta. However, from a fundamental point of view, rooting the product of two inequivalent theories is not in general expected to make sense.

The authors of Ref. [1] proceed to rewrite the partition function for their theory in terms of a partially quenched approach with three ghost fields. Again they do not distinguish the RCT Dirac operator used and assume they can use the same one for each field, including the ghosts. Actually, Ref. [3] also raises the possibility of using ghost fields to reduce the flavor content of unrooted staggered quarks, but emphasizes the important proviso that the ghosts must be formulated with a chiral operator, such as the overlap, to properly cancel the inequivalent tastes.

In summary, Ref. [1] confuses two different rooted theories, one of which is correct and the other not. The distinction is a strong inter-taste coupling that survives in the continuum limit and does not allow the effects of a single taste to be isolated. The successes of past

simulations do suggest that these effects can be small for some observables, but it is incorrect to claim that they go away in the continuum limit. Finally, I note that the argument that staggered simulations are much faster than alternative approaches has recently become moot [9].

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